X-Band Sampling by the Occultation Data Assembly

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Extraneous peaks appear in the power spectrum of X-band radio science data from the Occultation Data Assembly after it has been processed. The article shows that the cause lies in an incompatibility between the hardware implementation of Nyquist sampling and the software processing. This also shows that a forthcoming change in the hardware will eliminate the problem.

I. Introduction

The Occultation Data Assembly (ODA) was recently installed in the Deep Space Network's 64-meter subnet. The assembly digitizes and records the received carrier signal, both S- and X-band, from various spacecraft. Several sampled bandwidths and analogue-to-digital converter (ADC) assignments are available, and have been used for the Pioneer Venus Orbiter, and Voyager 1 and 2 Jupiter occultations.

The digitized data provide sufficient information for reconstruction of the carrier for investigation of relatively short-time scale effects. There is, however, an incompatibility between the way the X-band receiver is sampled in a commonly used mode, and the software which is used to reduce the X-band data. The problem is discussed below and examples are presented for the effects on a carrier signal, and on the filter bandpass characteristics.

II. The Problem

The ODA is commonly configured to sample the S-band receiver with one of its four ADCs, and sample the X-band receiver with the remaining three ADCs. The receiver bandwidths have a 1:3, S- to X-band ratio. All ADCs have equal sample frequencies, but the second and third X-band ADCs are staggered in time from the first. This is accomplished by multiplying the sampling frequency by 20, and sampling with the first ADC on the first cycle (simultaneous with the S-band ADC), the second ADC on the eighth cycle and finally, the third ADC on the fifteenth cycle. This gives X-band samples at 7/20, 7/20, 6/20 intervals with respect to the S-band samples. Thus, the X-band receiver is sampled irregularly, giving rise to extraneous power peaks when the data is Fourier transformed; these peaks are purely artifacts of the sampling process. Likewise, the filter bandpass characteristics are distorted by the sampling.

Although the waveform could be correctly reconstructed from the data by modification of the analysis software, the ODA hardware will be modified to sample regularly at S- and X-band. Currently, the X-band power spectrum obtained is, typically, as shown in Fig. 1, for a single-sampled signal. The relative amplitudes and frequencies of the sampling artifacts are computed below.

III. Calculations

The calculations that follow show first, how the signal can be reconstructed from its samples; second, the result if such a reconstruction process were applied to irregularly sampled waveforms; third, the transfer function for the "digital filter" formed by the ODA plus FFT (Fast Fourier Transform) software; and fourth, its effects on a nearly monochromatic input (Fig. 1).

Any waveform that is bandwidth limited may be represented by discrete samples if the samples are taken at a frequency of twice that bandwidth. The open loop receiver bandwidth limits the signal.

If $f(t)$ is the waveform, the sampled waveform can be represented by

$$f_s(t) = f(t) \sum_{n=-\infty}^{\infty} \delta(t - n\Delta T),\ \Delta T = \text{sample interval}$$

The Fourier transform of this is

$$F_s(\omega) = \frac{1}{2\pi} F(\omega) * \omega_0 \sum_{n=-\infty}^{\infty} \delta(\omega - n\omega_0),\quad \omega_0 = \frac{2\pi}{\Delta T}$$

Performing the convolution and substituting for ω_0,

$$F_s(\omega) = \frac{1}{\Delta T} \sum_{n=-\infty}^{\infty} F(\omega - n\omega_0)$$

which is just $F(\omega)$ repeated every ω_0 radians. If the bandwidth of the waveform is less than half the sample frequency, there is no overlapping of the terms of $F_s(\omega)$. $F(\omega)$ can be recovered from $F_s(\omega)$ by multiplying by a gate function

$$G_{\omega_0}(\omega) = \begin{cases} 1, & |\omega| \leqslant \omega_0/2 \\ 0, & |\omega| > \omega_0/2 \end{cases}$$

$$F(\omega) = \Delta T F_s(\omega) G_{\omega_0}(\omega)$$

or, in the time domain,

$$f(t) = \Delta T f_s(t) * g_{\omega_0}(t)$$

$$g_{\omega_0}(t) = \frac{\omega_0}{2\pi} \operatorname{sinc} \frac{\omega_0 t}{2},\quad \operatorname{sinc} x = \frac{\sin x}{x}$$

Thus $f(t)$ can be reconstructed from $f_s(t)$. Performing the convolution,

$$f(t) = \sum_{n=-\infty}^{\infty} f(n\Delta T) \operatorname{sinc} \frac{\omega_0}{2} (t - n\Delta T)$$

The irregularly sampled function, $\tilde{f}(t)$, can be written in terms of $f(t)$:

$$\begin{aligned} \tilde{f}(t) = \sum_{n=-\infty}^{\infty} \Big\{ & f(3n\Delta T) \operatorname{sinc} [t - 3n\Delta T]\, \omega_0/2 \\ & + f(3n\Delta T + \epsilon) \operatorname{sinc} [t - (3n+1)\Delta T]\, \omega_0/2 \\ & + f(3n\Delta T + 2\epsilon) \operatorname{sinc} [t - (3n+2)\Delta T]\, \omega_0/2 \Big\} \end{aligned}$$

In the X-band case, ϵ is 7 clock counts. Taking the transform,

$$\begin{aligned} \tilde{F}(\omega) = \frac{2\pi}{\omega_0} \sum_{n=-\infty}^{\infty} \Big\{ & f(3n\Delta T) \exp -i3n\Delta T\omega \\ & + f(3n\Delta T + \epsilon) \exp -i(3n+1)\,\Delta T\omega \\ & + f(3n\Delta T + 2\epsilon) \exp \\ & -i(3n+2)\Delta T\omega \Big\},\quad |\omega| < \frac{\omega_0}{2} \end{aligned}$$

This may be rewritten in terms of $F(\omega)$. Consider:

$$f_a(t) = \sum_{n=-\infty}^{\infty} f(t + a\epsilon)\, \delta(t - 3n\Delta T)$$

$$\begin{aligned} F_a(\omega) &= \frac{1}{2\pi} F(\omega) \exp ia\epsilon\omega * \frac{2\pi}{3\Delta T} \sum_{n=-\infty}^{\infty} \delta\left(\omega - \frac{2\pi n}{3\Delta T}\right) \\ &= \sum_{-\infty}^{\infty} \frac{1}{3\Delta T} F\left(\omega - \frac{n\omega_0}{3}\right) \exp ia\epsilon\left(\omega - \frac{n\omega_0}{3}\right) \end{aligned}$$

Using an equivalent form of $f_a(t)$,

$$f_a(t) = \sum_{n=-\infty}^{\infty} f(3n\Delta T + a\epsilon)\,\delta\,(t - 3n\Delta T)$$

$$F_a(\omega) = \sum_{n=-\infty}^{\infty} f(3n\Delta T + a\epsilon) \exp - 3in\Delta T\omega$$

Multiplying both expressions for $F_a(\omega)$ by $(2\pi/\omega_0)$ exp $-ia\Delta T\omega$,

$$\frac{2\pi}{\omega_0} \sum_{n=-\infty}^{\infty} f(3n\Delta T + a\epsilon) \exp -i\,(3n + a)\,\Delta T\omega =$$

$$\frac{1}{3} \sum_{n=-\infty}^{\infty} F\left(\omega - \frac{n\omega_0}{3}\right) \exp ia\omega\,(\epsilon - \Delta T) \exp -ia\epsilon\frac{n\omega_0}{3}$$

Taking a to be 0, 1, or 2 gives all the terms of $\widetilde{F}(\omega)$:

$$\widetilde{F}(\omega) = \frac{1}{3} \sum_{n=-\infty}^{\infty} F\left(\omega - \frac{n\omega_0}{3}\right) (1 + \exp iQ$$

$$+ \exp 2iQ), \quad |\omega| < \frac{\omega_0}{2}$$

where $Q = \omega(\epsilon - \Delta T) - \frac{n}{3}\omega_0\epsilon$. Note that $\widetilde{F}(\omega) = F(\omega)$ for $\epsilon = \Delta T$.

IV. Conclusion

A. Effect on a Monochromatic Signal

A typical power spectrum at X-band is shown in Fig. 1. For the purposes of this study, the received signal may be approximated by a delta function.

$$F(\omega) = \delta\left(\omega - \frac{\omega_0}{4}\right)$$

yields

$$\widetilde{F}(\omega) = A_0\,\delta\left(\omega - \frac{\omega_0}{4}\right) + A_1\,\delta\left(\omega + \frac{\omega_0}{12}\right)$$

$$+ A_2\,\delta\left(\omega + \frac{5\omega_0}{12}\right)$$

where the amplitudes A_0, A_1, A_2 are determined by

$$A_{-n} = \frac{1}{3}\left\{1 + \cos\left[\omega(\epsilon - \Delta T) - \frac{n}{3}\omega_0\epsilon\right]\right.$$

$$\left. + \cos 2\left[\omega(\epsilon - \Delta T) - \frac{n}{3}\omega_0\epsilon\right]\right\}$$

The A_1 and A_2 terms represent the extraneous power peaks. Physically, the components at frequencies $-\omega_0/12$ and $-5\omega_0/12$ may be considered to be at $\omega_0/12$ and $5\omega_0/12$. This is so because the power at a frequency is proportional to the square of the amplitude of that frequency component. Thus,

$$[A \cos(-\omega)]^2 = A^2 \cos^2(-\omega) = (A \cos \omega)^2$$

and

$$[A \sin(-\omega)]^2 = A^2 \sin^2(-\omega) = (A \sin \omega)^2$$

so that any component at $-\omega$ may be considered to be at $+\omega$.

B. Effect on Transfer Curve

The initial bandwidth limiting that occurs in the open loop receiver gives a transfer curve similar to that shown in Fig. 2. The equation for $\widetilde{F}(\omega)$ has been computed for that transfer function to give the new transfer curve. The change in power in dB is plotted in Fig. 3 for the following case:

$\Delta T = 3.33 \times 10^{-5}$ s

$\omega_0 = 188496$ rad/s = 30 KHz

$\epsilon = 3.5 \times 10^{-5}$ s

$$\text{power change} = 20 \log \frac{\widetilde{F}(\omega)}{F(\omega)}$$

The transfer of the signal across the digital filter is nearly linear (deviations around 1 dB).

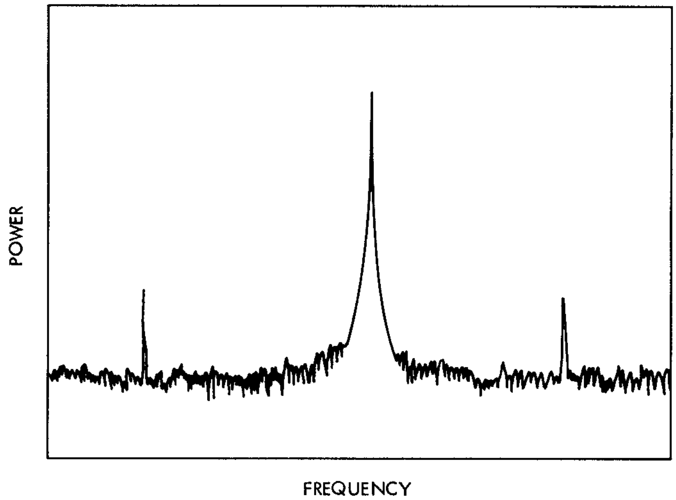

Fig. 1. X-band power spectrum from ODA

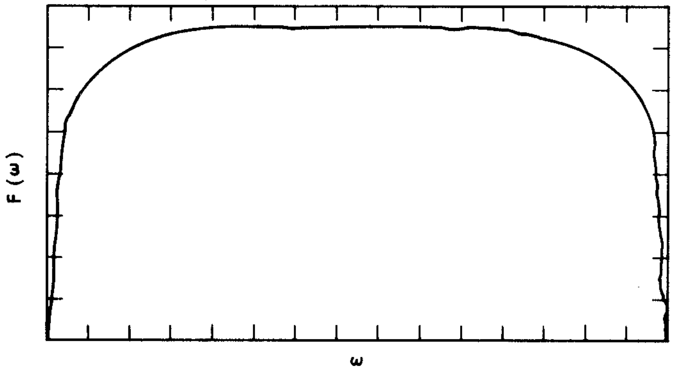

Fig. 2. Open loop receiver transfer curve

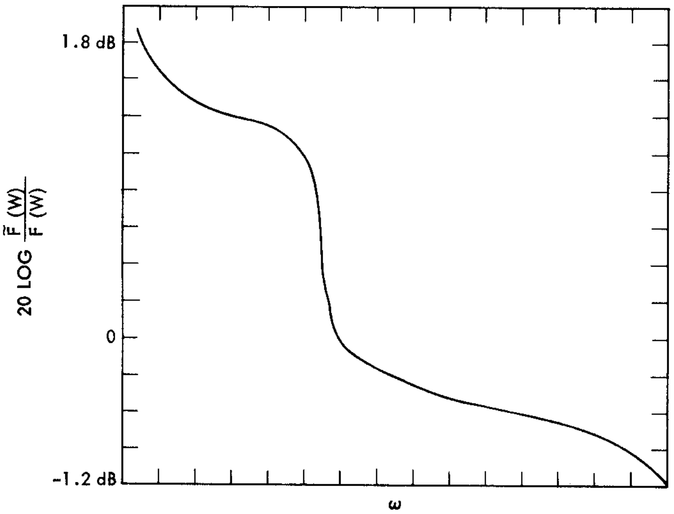

Fig. 3. Difference in signal power between ODA with regular sampling and ODA with irregular sampling